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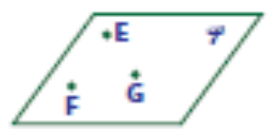
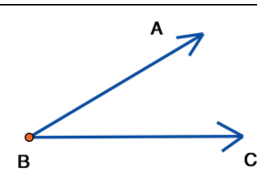
Geometry

Name: ______________________________ Date: _____________ Per.: ________

FINAL EXAM REVIEW – Basic Geometry Vocabulary

Euclid:
- the father of Geometry
- studied by Abraham Lincoln
- built an *axiomatic* system of Geometry
  - based on **axioms** – statements accepted as true
  - ex: A straight line segment can be drawn joining any two points.

	Description	Figure	Symbol
point*	describes a location; zero dimensions		P or Point P
line*	a collection of points along a straight path with no endpoints; one dimension (length)		$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
plane*	a flat surfaces that extends indefinitely; two dimensions (length and width)		Plane EFG or Plane *T*
ray	a collection of points along a straight path with one endpoint which extends indefinitely in one direction; one dimension (length)		$\overrightarrow{PQ}$
line segment	a collection of points along a straight path with two endpoints; one dimension (length) **measurable*		$\overline{XY}$ or $\overline{YX}$
angle	two rays that meet at a point (this point is the **vertex**) **measurable*		$\angle ABC$

*__undefined__ terms of Geometry… MANY definitions have their roots in these three words

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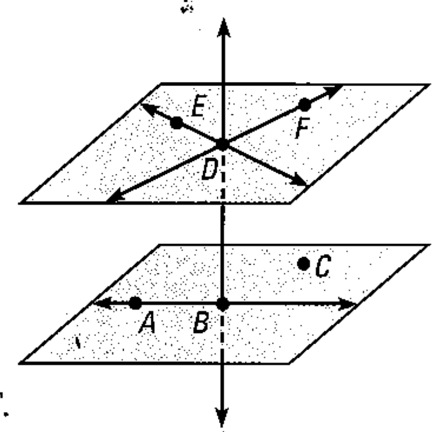
Directions: I HIGHLY recommend answering these on a separate sheet of paper.

1. Describe what each of these symbols means: $\overline{PQ}$, $\overrightarrow{PQ}$, $\overleftrightarrow{PQ}$, $\overline{QP}$.

2. Sketch a line that contains point R between points S and T. Which of the following are true?

   A. $\overrightarrow{SR}$ is the same as $\overrightarrow{ST}$.
   B. $\overleftrightarrow{SR}$ is the same as $\overleftrightarrow{RT}$.
   C. $\overrightarrow{RS}$ is the same as $\overrightarrow{TS}$.
   D. $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are opposite rays.
   E. $\overline{ST}$ is the same as $\overline{TS}$.
   F. $\overrightarrow{ST}$ is the same as $\overrightarrow{TS}$.

Decide whether the statement is *true* or *false*.



3. Points A, B, and C are collinear.
4. Points A, B, and C are coplanar.
5. Point F lies on $\overleftrightarrow{DE}$.
6. $\overleftrightarrow{DE}$ lies on plane DEF.
7. $\overleftrightarrow{BD}$ and $\overleftrightarrow{DE}$ intersect.
8. $\overleftrightarrow{BD}$ is the intersection of plane ABC and plane DEF.

Short Answer:

1. What are the undefined terms?

2. Why will two points ALWAYS be collinear? Why will three points always be coplanar?

3. In what way(s) was Euclid influential?

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FINAL EXAM REVIEW – Constructions

Perpendicular Bisector: http://www.mathopenref.com/constbisectline.html

- Given line segment AB
- Draw circle A (this means the center is A) with radius AB OR Draw circle A with radius of over half of AB
- Draw circle B (this means the center is B) with the same radius
- Label the two intersection points C and D
- Draw a line between C and D

Angle Bisector: http://www.mathopenref.com/constbisectangle.html

- Set the compass to any width
- Draw an arc with center A (vertex) (or draw a circle with center A) so that it intersects with both sides of the angle
- Label the intersection points as B and C (these are the points where the arc intersected the angle)
- Draw an arc with center B/draw a circle with center B
- Draw an arc with center C with the compass set to the same width (set to the same radius)
- Label the intersection point D
- Draw a ray/line/line segment from A to D

Equilateral Triangle: http://www.mathopenref.com/constequilateral.html

- Given line segment AB
- Draw circle A with radius AB
- Draw circle B with radius AB
- Mark one intersection point of the circles as C
- Create line segments AC and BC

WHY DO THEY WORK?

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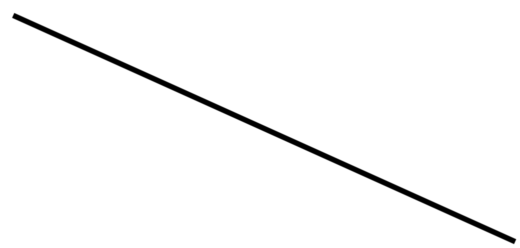
You can do these on a separate sheet of paper by drawing a line segment with a straight edge.

1. Construct the perpendicular bisector of $\overline{DE}$. When complete, mark the congruent segments and right angles.



2. Construct an equilateral triangle.

3. How does the construction of an equilateral triangle ensure that all sides are congruent?

4. How does the construction of an angle bisector ensure that two congruent angles are created?

** a similar question regarding perpendicular bisectors will be later in the review **

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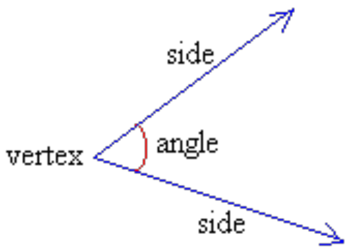
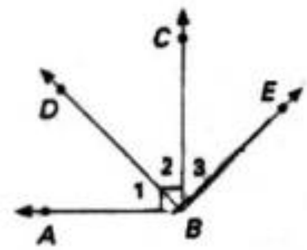
FINAL EXAM REVIEW – Angle Relationships and Angle Proofs

What is an angle?



*the angle is the space between the two sides… a portion of 360°

How do you name an angle?

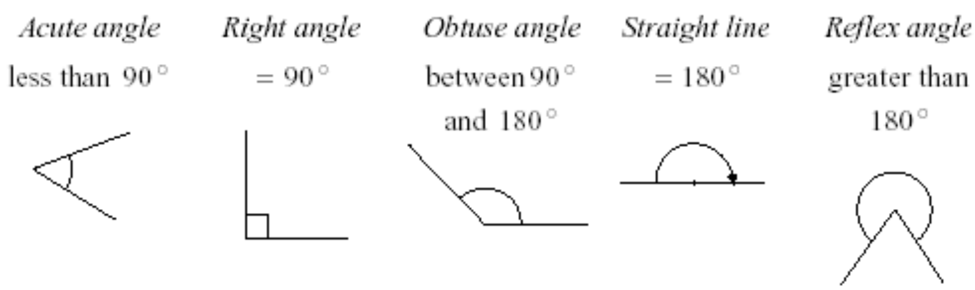


$\angle 1$ can also be named $\angle ABD$ or $\angle DBA$

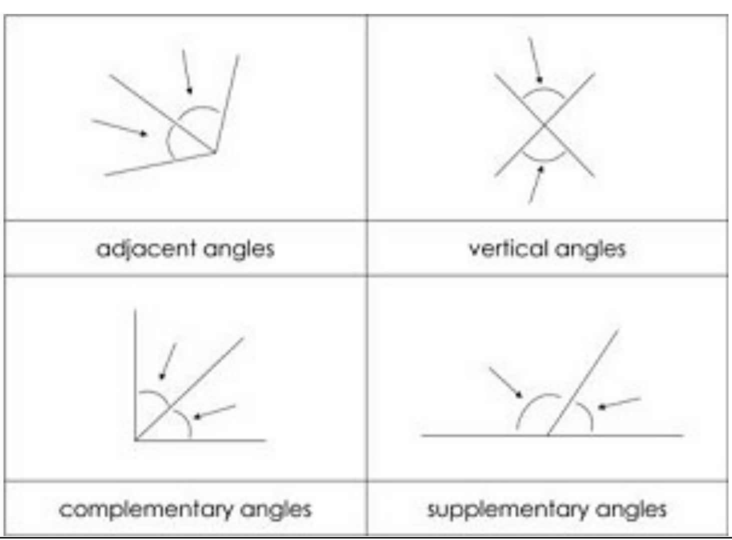
$\angle 2$ can also be named $\angle DBC$ or $\angle CBD$

notice that the VERTEX is always the letter in the middle

How can you describe an angle?



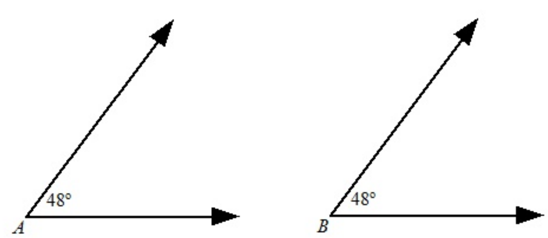
Angle relationships



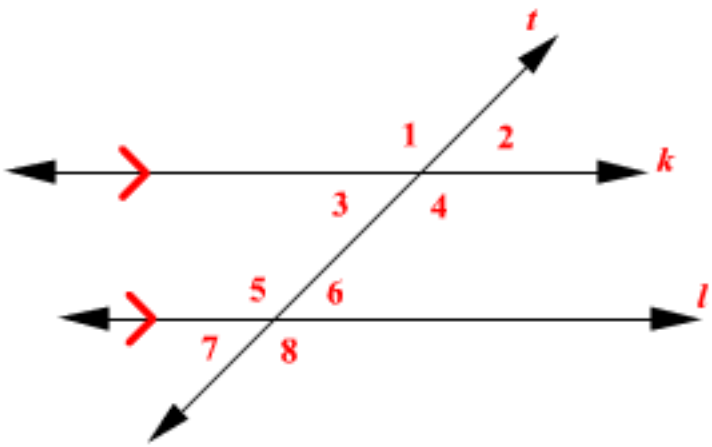
Congruent Angles

Congruent angles have the same angle measure.

$\angle A \cong \angle B$ because the measure of both angles is 48°.



Parallel Lines cut by a Transversal



In the diagram to the left, line *l* and line *k* are parallel. Line *t* is a **transversal** because it cuts through both lines.

Key Points:

1. Vertical angles are congruent.
Ex: $\angle 1 \cong \angle 4$ and $\angle 7 \cong \angle 6$

2. Alternate interior angles are congruent.
Ex: $\angle 3 \cong \angle 6$ and $\angle 5 \cong \angle 4$

3. Corresponding angles are congruent.
Ex: $\angle 1 \cong \angle 5$ and $\angle 8 \cong \angle 4$

4. Same side interior angles are supplementary.
Ex: $m\angle 6 + m\angle 4 = 180°$

5. Same side exterior angles are supplementary.
Ex: $m\angle 8 + m\angle 2 = 180°$

6. Linear pairs are supplementary.
Ex: $m\angle 1 + m\angle 2 = 180°$

- How can we prove lines are parallel?
  - If alternate interior, alternate exterior, or corresponding angles are congruent.
  - If same side interior or same side exterior angles are supplementary.

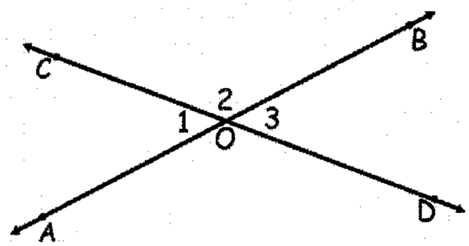
- If we are given that lines are parallel, we can state that alternate interior or exterior angles and corresponding angles are congruent and same side interior or exterior angles are supplementary if the lines are parallel.

- Corresponding Angles Postulate: When parallel lines are cut by a transversal, corresponding angles are congruent. This is a **postulate** – something accepted as true without proof – so this is **very** helpful to use in proofs.

- Corresponding Angles CONVERSE: When corresponding angles are congruent, the lines are parallel (*simplified version*).

Theorem: Vertical angles are congruent.

Given: $\angle AOB$ and $\angle COD$ are straight angles
Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
① $\angle AOB$ and $\angle COD$ are straight angles	① Given
② $\angle 1$ and $\angle 2$ are a linear pair $\angle 2$ and $\angle 3$ are a linear pair	② They are both pairs of adjacent, supplementary angles (def. of linear pair)
③ $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	③ If two angles form a linear pair, then they are supplementary
④ $m\angle 1 + m\angle 2 = 180°$ $m\angle 2 + m\angle 3 = 180°$	④ If two angles are supplementary, then they sum to 180° (def. of supplementary)
⑤ $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	⑤ substitution prop.
⑥ $m\angle 1 = m\angle 3$	⑥ subtraction prop.
⑦ $\angle 1 \cong \angle 3$	⑦ angles that have equal measure are congruent (def. of congruent angles)

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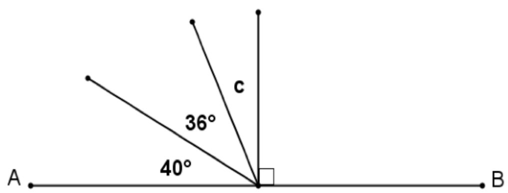
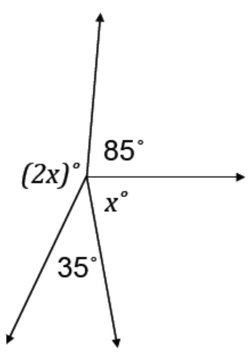
FINAL EXAM REVIEW – Angle Relationships and Angle Proofs

Directions: Find the value of each "lettered" angle. **Explain your step-by-step process.** For example: *Angle b is 50 degrees because it is part of a linear pair.*

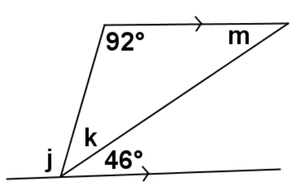
1.



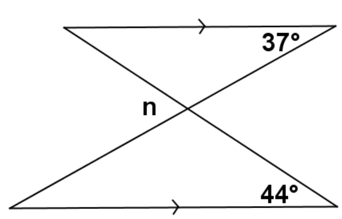
2.



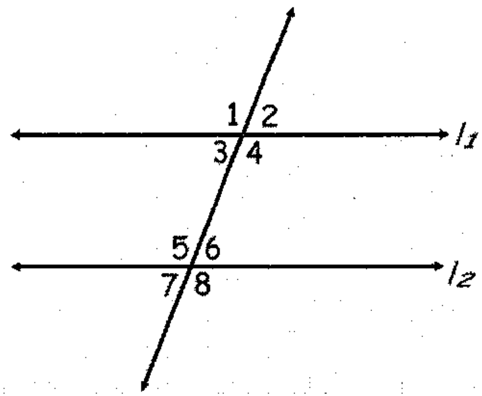
3.



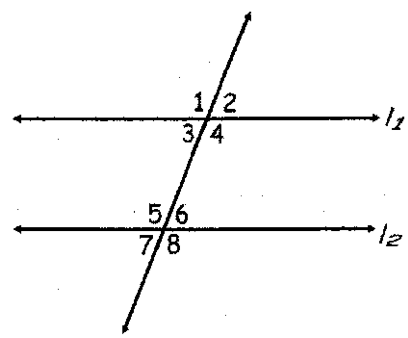
4.



Given: $l_1 \parallel l_2$
Prove: $\angle 3 \cong \angle 6$

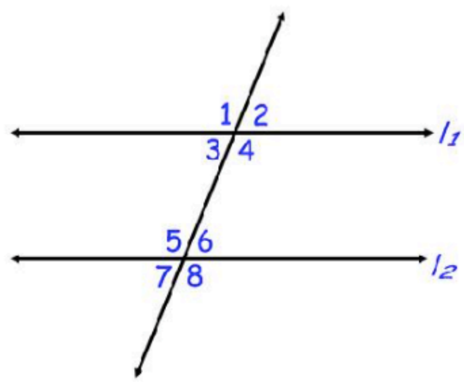


Given: $l_1 \parallel l_2$
Prove: $\angle 1$ is supplementary to $\angle 7$



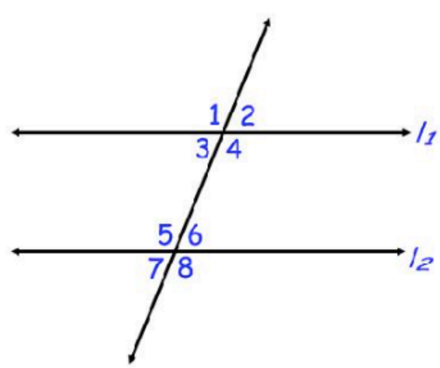
Given: $\angle 3 \cong \angle 6$

Prove: $l_1 \parallel l_2$



Given: $\angle 3$ and $\angle 5$ are supplementary

Prove: $l_1 \parallel l_2$



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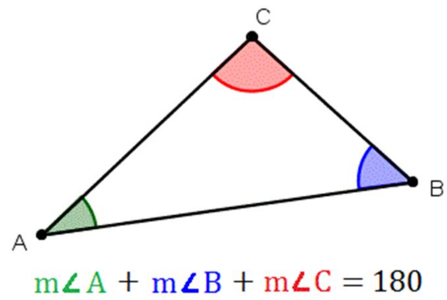
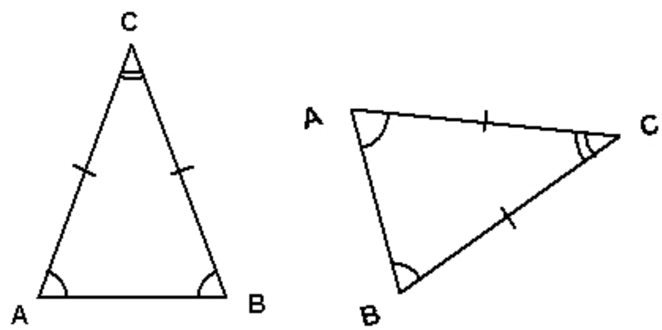
FINAL EXAM REVIEW – Properties of Triangles

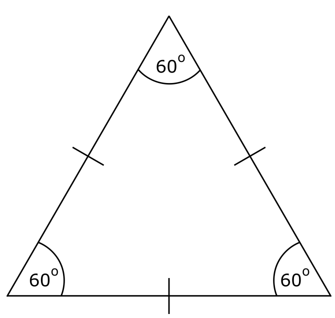
1. The sum of the interior angles of any triangle is 180 degrees.



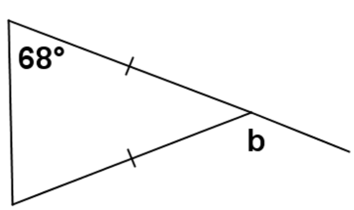
2. The base angles of an isosceles triangle are congruent to each other.



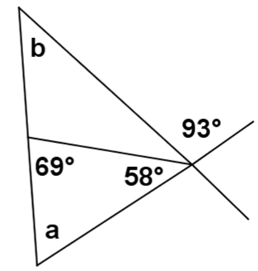
3. All interior angles of an equilateral triangle are 60 degrees.



Ex:

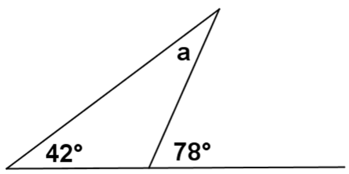


1. Find the measures of angles a and b in the figure to the right. Justify your results.



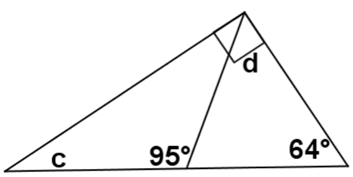
In each figure, determine the measures of the unknown (labeled) angles. Give reasons for your calculations.

2.

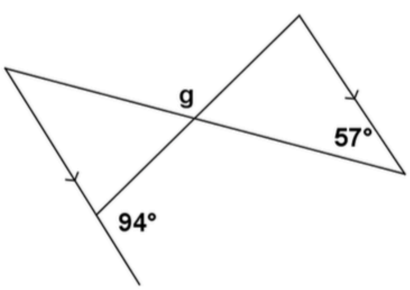


$m\angle a =$ ____________________

3.



4.



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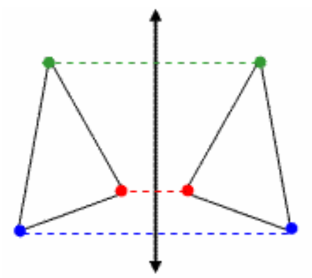
FINAL EXAM REVIEW – Transformations

- A **transformation** is a **function**: There is a set of points (the pre-image), which is the input. We apply a rule (for example, translate right 4 and up 6) and then there is a new location for the set of points (the image), which is the output.

- There are three rigid motions: rotations, reflections, translations

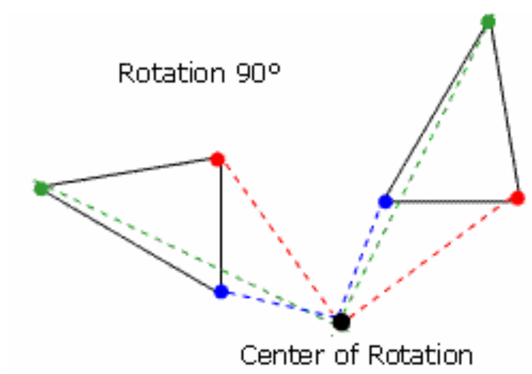
- A **rigid motion** maps the pre-image onto the image. It preserves angle measurements and distances; therefore, the image and pre-image are **congruent** after a series of rigid motions.

- **Reflection**: corresponding points of the pre-image and image are equal distances from the line of reflection/line of symmetry

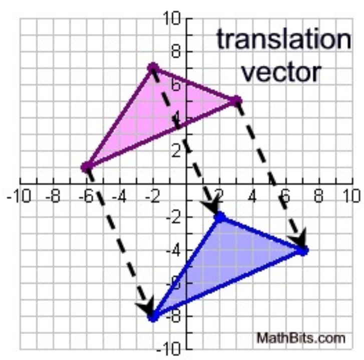


All points on a perpendicular bisector are equidistant from the endpoints of the segment being bisected.

- **Rotation**: corresponding points of the pre-image and image are equal distances from the center of rotation



- **Translation**: corresponding points of the pre-image and image are equal distances from each other along the same vector



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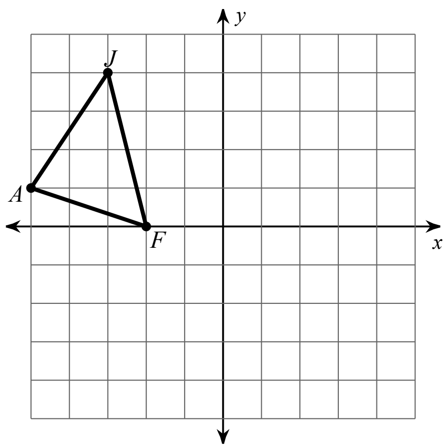
1. Rotate the figure $-90°$ about the center P$(-2,-2)$. Label the resulting image appropriately.



Indicate which statements are true:

A. ΔAJF and $\Delta A'J'F'$ have the same perimeter

B. ΔAJF and $\Delta A'J'F'$ have the same area

C. $m < A = m < A'$

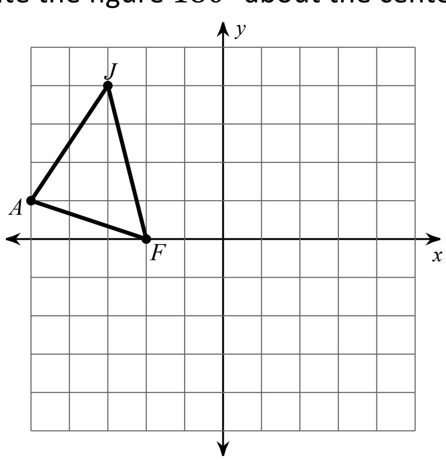
D. F and F' have the same location

E. JF = J'F'

F. FF' = JJ'

G. FP = F'P

2a. Rotate the figure $180°$ about the center P$(-3,1)$. Label the resulting image appropriately.



2b. What is the measure of $\angle FPF'$?

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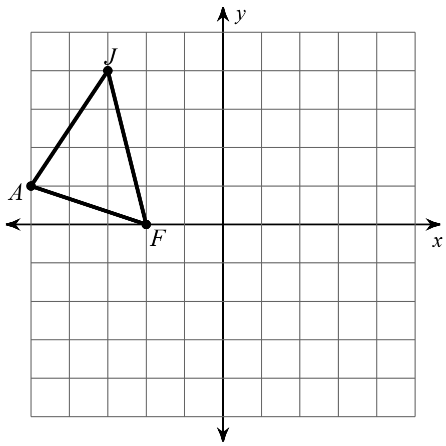
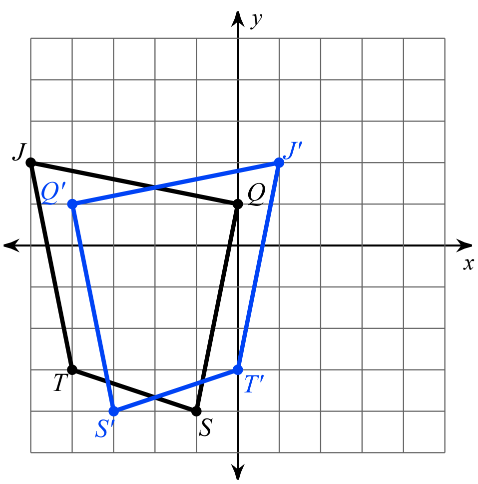
3. Reflect the figure across $y = x$. Label the image appropriately. (*3 pts.*)



4. There is a reflection that transforms JQST to J'Q'S'T'. Draw the line of reflection **and** write the equation of the line. Then, describe **why** that is the line of reflection.



5. Describe how a transformation is a function.

6. Name three rigid motions and describe how they are related to congruence?

7. Describe how a perpendicular bisector relates to the line of symmetry in a reflection. Based on what you know about perpendicular bisectors, how do you know all corresponding points are equidistant from the line of symmetry?

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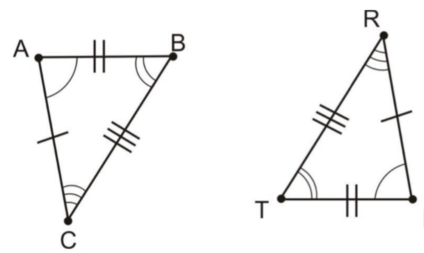
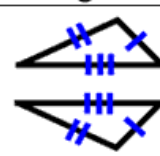
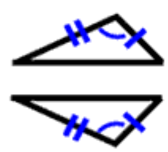
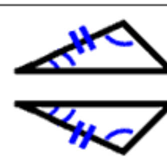
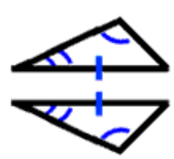
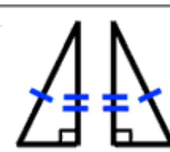
FINAL EXAM REVIEW – Triangle Proofs

- Two triangles are congruent if all **corresponding sides** are the **same length**, and if all **corresponding angles** have the **same measure**.

- Both pairs of triangles below are congruent. You can tell the first pair is congruent because corresponding sides and angles are the same measure. You can tell the second pair is congruent because of the congruence markings.



- Shortcuts for proving triangles congruent:

Congruence	Explanation	Diagram
SSS	When two triangles have three corresponding sets of sides congruent, use SSS to say the triangles are congruent.	
SAS	When two triangles have two pairs of sides congruent and the angles between them are congruent, use SAS to say the triangles are congruent.	
ASA	When two triangles have two pairs of angles congruent and the sides between them are congruent, use ASA to say the triangles are congruent.	
AAS	When two triangles have two pairs of angles congruent and the sides not between them are congruent, use AAS to say the triangles are congruent.	
HL	When to right triangles have congruent hypotenuses and a pair of congruent legs, use HL to say the triangles are congruent.	

CAREFUL! AAA (Angle Angle Angle) and **SSA** (Side Side Angle) do **NOT** prove two triangles congruent! DO NOT USE THEM IN A PROOF FOR TRIANGLE CONGRUENCE!

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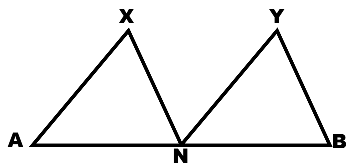
- Important Concepts for Proofs

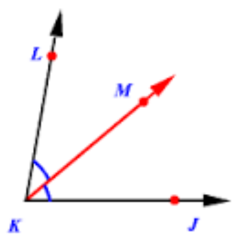
  - Midpoint: the middle point of a line segment; It is equidistant from both endpoints; it bisects the segment.



In the diagram to the left:

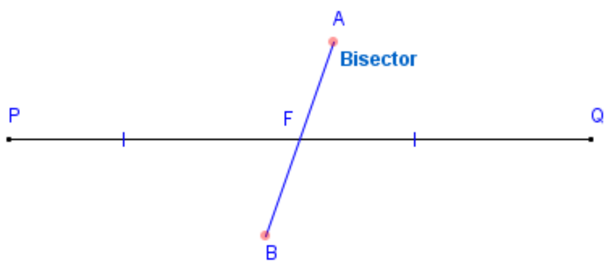
N **is the midpoint of** $\overline{AB}$ **so…**
$\overline{AN} \cong \overline{BN}$

  - Bisector: a line that cuts an angle or line segment into two equal parts



In the diagram to the left:

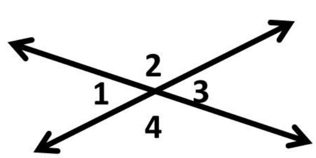
$\overrightarrow{KM}$ **bisects** $\angle LKJ$ **so…** $\angle LKM \cong \angle JKM$



In the diagram to the left:

$\overline{AB}$ **bisects** $\overline{PQ}$ **so…** $\overline{PF} \cong \overline{QF}$

  - Vertical angles



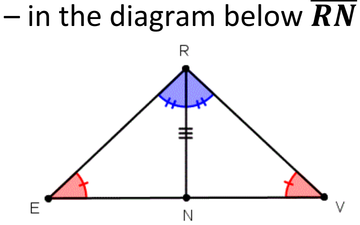
In the diagram to the left:

Angles 1 and 3 are vertical angles so they are CONGRUENT.

Angles 2 and 4 are vertical angles so they are CONGRUENT.

  - Corresponding Parts of Congruent Triangles are Congruent (CPCTC): If two triangles are congruent, then the corresponding parts of those triangles are congruent!

  - Reflexive Property – in the diagram below $\overline{RN} \cong \overline{RN}$ by the reflexive property



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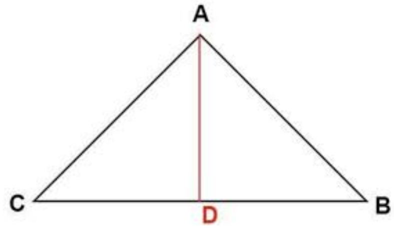
Geometry

Directions: Use the diagram to answer the questions below.



$\overline{AD}$ bisects $\overline{CB}$; therefore, $\overline{CD} \cong \overline{BD}$. You are asked to prove $\triangle ADC \cong \triangle ADB$.

a. To prove congruence by SSS, what two additional congruence statements are needed?

1. 2.

b. To prove congruence by SAS, what two additional congruence statements are needed?

1. 2.

c. To prove congruence by ASA, what two additional congruence statements are needed?

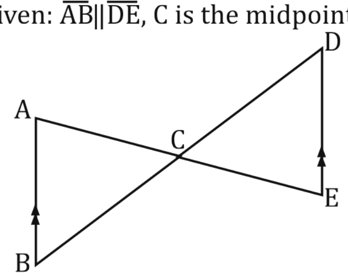
1. 2.

d. To prove congruence by AAS, what two additional congruence statements are needed?

1. 2.

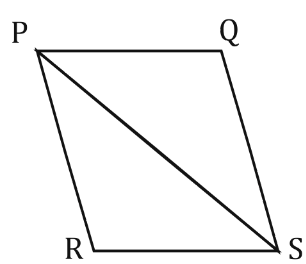
COMPLETE ON A SEPARATE SHEET OF PAPER

Given: $\overline{AB} \| \overline{DE}$, C is the midpoint of $\overline{AE}$

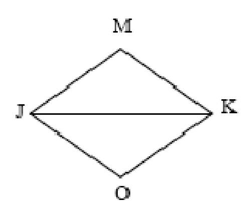


Prove: $\overline{BC} \cong \overline{DC}$

Given: PQRS is a parallelogram



Prove: $\triangle RPS \cong \triangle QSP$



Given: $\overline{MK} \cong \overline{OK}$
$\overline{KJ}$ bisects $\angle MKO$
Prove: $\overline{KJ}$ bisects $\angle MJO$

FINAL EXAM REVIEW – Properties of Parallelograms

Parallelogram

1. Both pairs of opposite sides are parallel.
2. Both pairs of opposite sides are congruent.
3. One pair of opposite sides are parallel and congruent.
4. Diagonals bisect each other.
5. Both pairs of opposite angles are congruent.
6. Consecutive angles are supplementary.

Rectangle

1. All the properties of a parallelogram.
2. Has a right angle.
3. Diagonals are congruent.

Rhombus

1. All the properties of a parallelogram.
2. All sides are congruent.
3. Diagonals are perpendicular.
4. Diagonals bisect the opposite angles.

Square

1. All the properties of a parallelogram.
2. All the properties of a rectangle.
3. All the properties of a rhombus.

definition of a parallelogram: quadrilateral with opposite sides that are parallel

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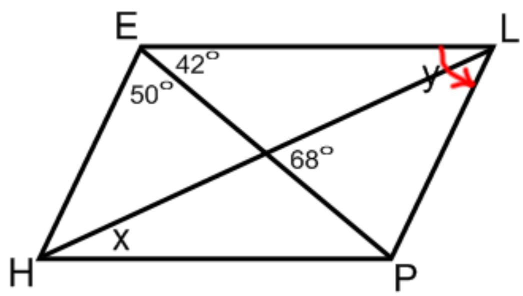
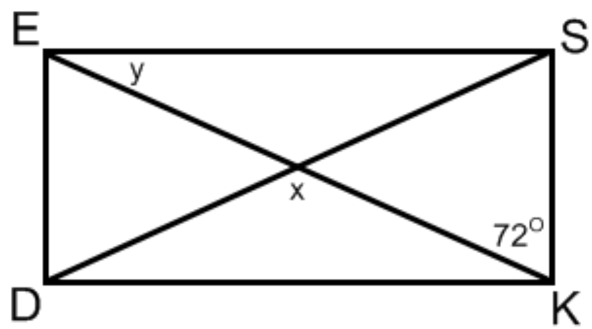
Geometry

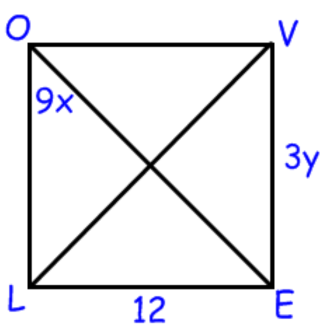
1. HELP is a parallelogram. Find the values of x and y:



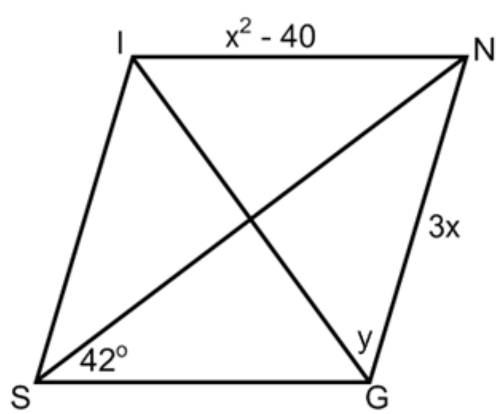
2. DESK is a rectangle. Find the values of x and y:



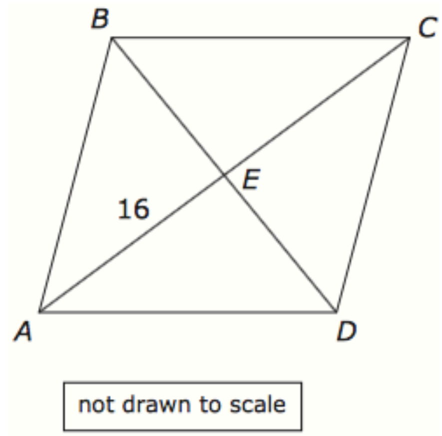
3. LOVE is a square. Find the values of x and y:



4. SING is a rhombus. Find the values of x and y:



The figure shows parallelogram *ABCD* with $AE = 16$.



14. Part A

Let $BE = x^2 - 48$ and let $DE = 2x$. What are the lengths of $\overline{BE}$ and $\overline{DE}$? Justify your answer.

Enter your answer and your justification in the space provided.

Part B

What conclusion can be made regarding the specific classification of parallelogram *ABCD*? Justify your answer.

Enter your answer and your justification in the space provided.

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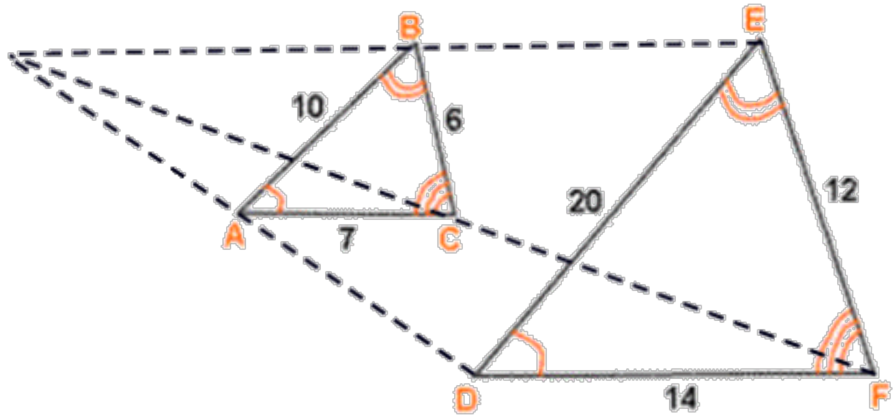
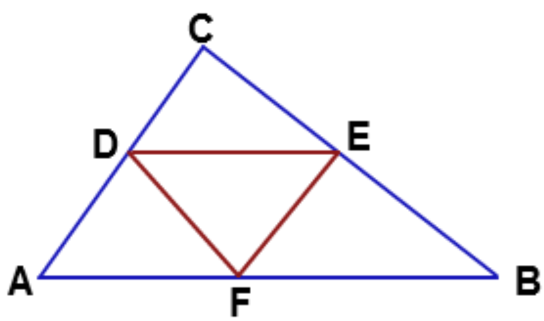
Geometry

FINAL EXAM REVIEW – Similarity (including midsegments and dilations)

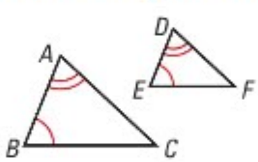


Midsegment Theorem: A midsegment connecting two sides of a triangle is parallel to the third side and is half as long.



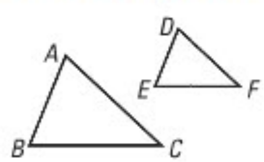
Shortcuts to proving triangles similar:

AA Similarity Postulate



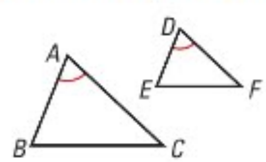
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

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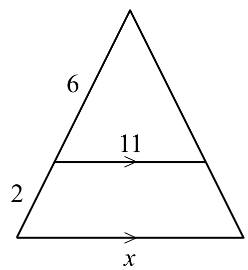
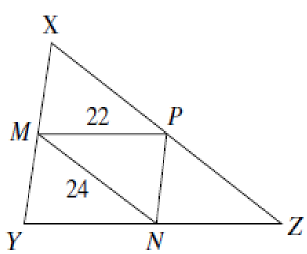
Geometry

1. Explain why the triangles are similar. Then find the value of *x*.



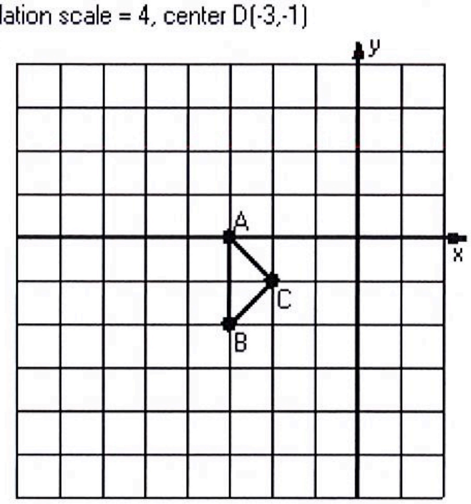
Not drawn to scale

2. M, P, and N are midpoints. Given triangle MNP has a perimeter of 60 cm, what is the perimeter of XYZ?

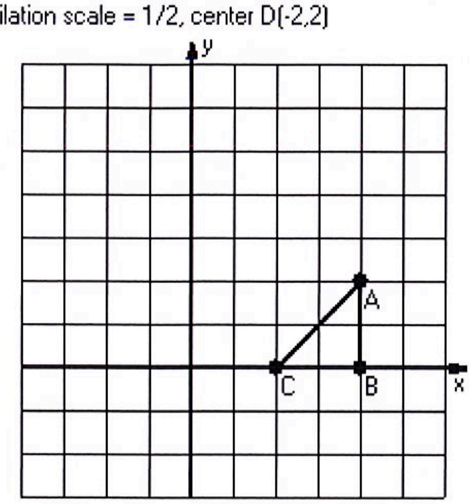


Directions: Perform the dilation according to the scale factor and center of dilation.

3) Dilation scale = 4, center D(-3,-1)



4) Dilation scale = 1/2, center D(-2,2)



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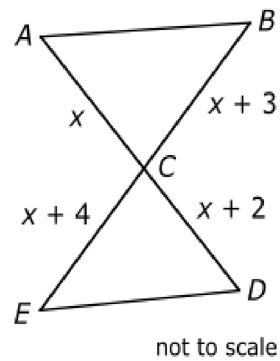
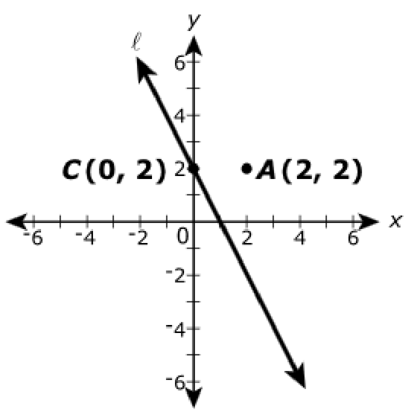
Geometry
5.

In the figure shown, the lengths of segments *AC*, *BC*, *CD*, and *CE* are given in terms of the variable *x*.



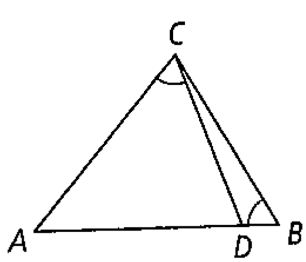
If $\overline{AB} \parallel \overline{DE}$, are the dimensions reasonable? Justify your answer.

6. In the coordinate plane shown, line *l* passes through point C and has a slope of -2. A dilation of line *l* with center *A* and a scale factor of 3 will produce a new line through point *C'*, the image of point *C*, with coordinates (,   ) and a slope of ________ .

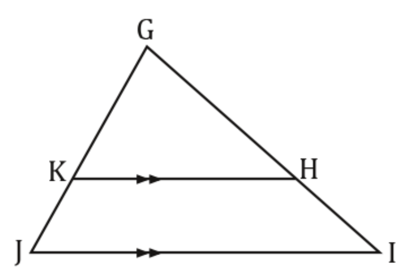


7. Complete the proofs on a separate sheet of paper.

Given: $\angle ABC \cong \angle ACD$
Prove: $\triangle ABC \sim \triangle ACD$



Given: JI||KH



Prove: $\triangle JGI \sim \triangle KGH$

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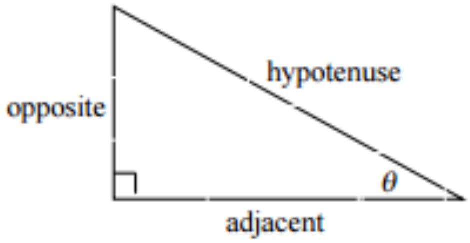
Geometry

FINAL EXAM REVIEW – Trigonometry

- We will focus on **right triangle trigonometry**... so we can only work with **right triangles**.
- There are **three** trigonometric functions we have worked with. Each is a **ratio**.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Inverse trigonometric functions:

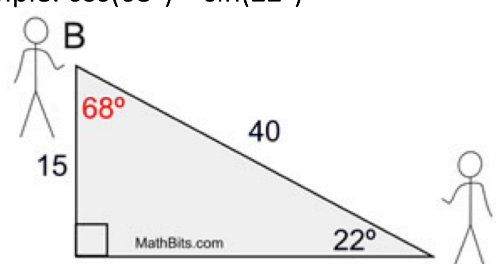
$\theta = \cos^{-1}(x) \quad \Leftrightarrow \quad x = \cos(\theta)$

$\theta = \sin^{-1}(x) \quad \Leftrightarrow \quad x = \sin(\theta)$

$\theta = \tan^{-1}(x) \quad \Leftrightarrow \quad x = \tan(\theta)$

- Complementary Angles:
  - $\sin\theta = \cos(90^\circ - \theta)$
  - $\cos\theta = \sin(90^\circ - \theta)$

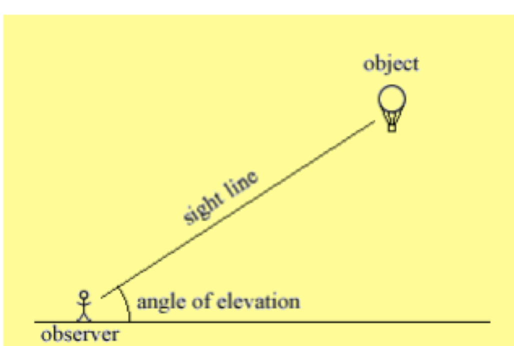
Example: $\cos(68^\circ) = \sin(22^\circ)$

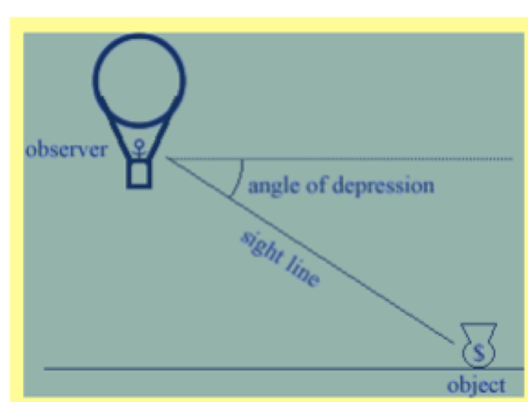


- Word Problems:

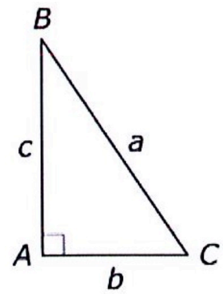
Angle of Elevation – The angle formed by the horizontal and the line of sight above the horizontal (to see an object that is higher than the observer).

Angle of Depression – The angle formed by the horizontal and the line of sight below the horizontal (to see an object that is lower than the observer).





10. The figure shows right $\triangle ABC$.

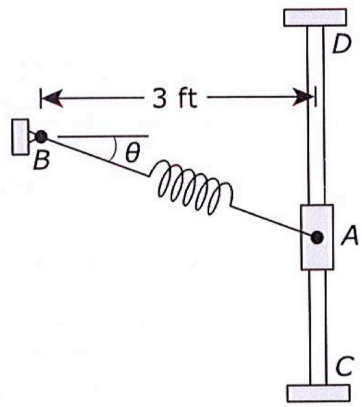


Which of the listed values are equal to the sine of B?

Select **all** that apply.

- [ ] A. $\frac{b}{c}$
- [ ] B. $\frac{c}{a}$
- [ ] C. $\frac{b}{a}$
- [ ] D. the cosine of B
- [ ] E. the cosine of C
- [ ] F. the cosine of $(90^\circ - B)$
- [ ] G. the sine of $(90^\circ - C)$

A spring is attached at one end to support *B* and at the other end to collar *A*, as represented in the figure. Collar *A* slides along the vertical bar between points *C* and *D*. In the figure, the angle θ is the angle created as the collar moves between points *C* and *D*.



When the spring is stretched and the distance from point *A* to point *B* is 5.2 feet, what is the value of θ to the nearest tenth of a degree?

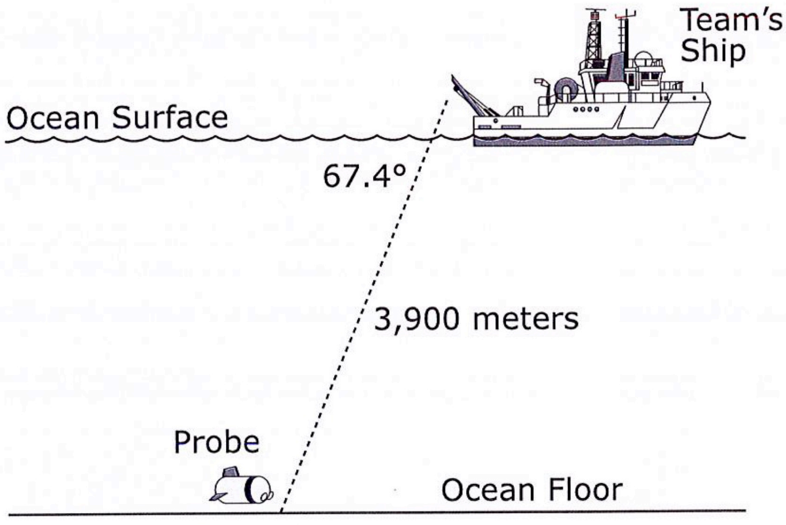
A. 35.2°

B. 45.1°

C. 54.8°

D. 60.0°

An archaeological team is excavating artifacts from a sunken merchant vessel on the ocean floor. To assist the team, a robotic probe is used remotely. The probe travels approximately 3,900 meters at an angle of depression of 67.4 degrees from the team's ship on the ocean surface down to the sunken vessel on the ocean floor. The figure shows a representation of the team's ship and the probe.



not to scale

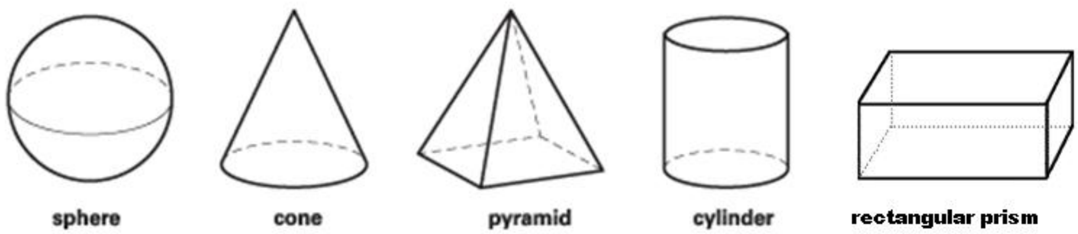
How many meters below the surface of the ocean will the probe be when it reaches the ocean floor?

Give your answer to the nearest hundred meters. Enter your answer in the box.

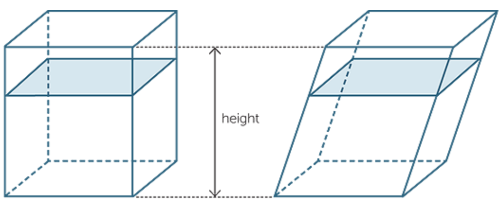
WLPCS
Geometry

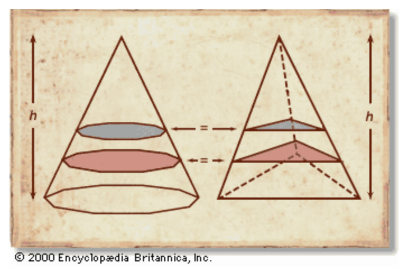
FINAL EXAM REVIEW – Area and Volume

- Geometric solids:

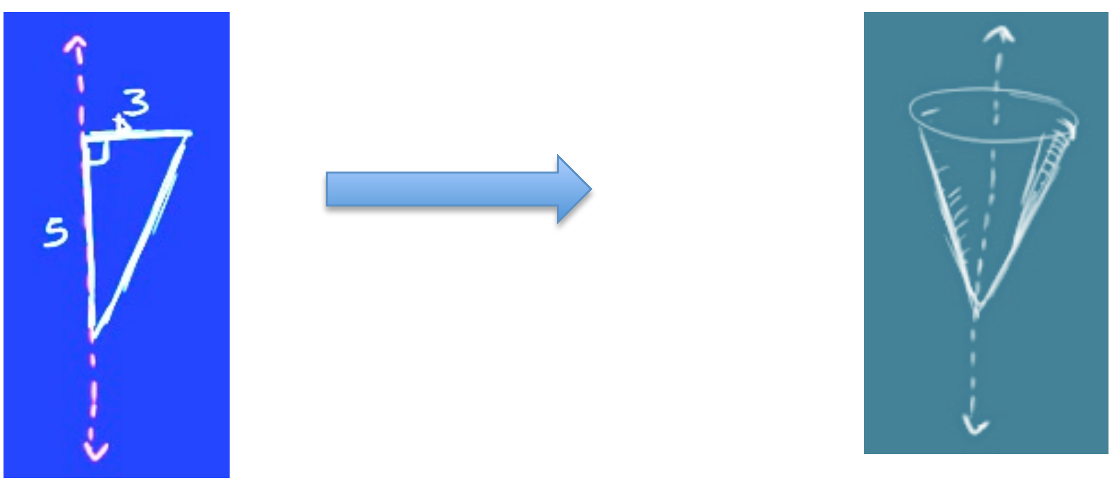


- Cavalieri's Principle: If two 3-D solids have the same height and cross sections of equal areas, then the two solids will have equal volume.





- Rotating a 2-D Shape to Form a 3-D Solid



- Cross-section: the 2-D figure formed by the intersection of a plane and a 3-D solid

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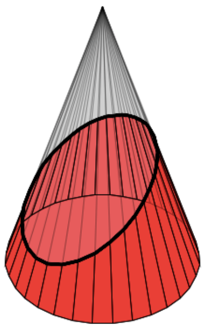
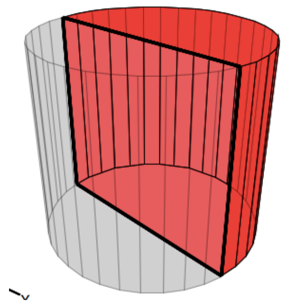
Geometry

Directions: Identify the 3-D solid AND the cross-section.

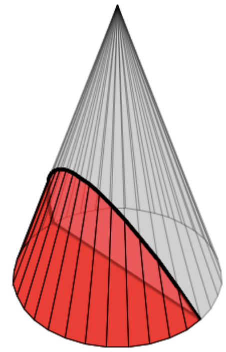
1.

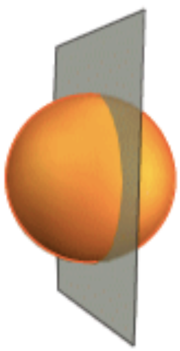


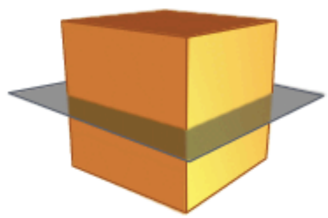
2.



3.

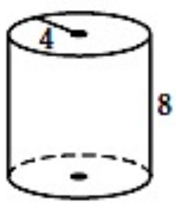




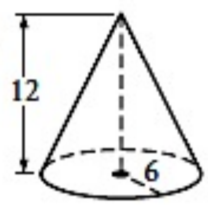


Directions: Find the volume of each A and B pair. Which is bigger?

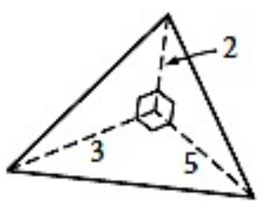
A.



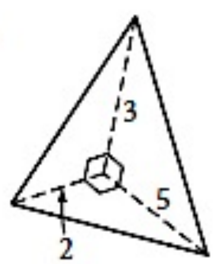
B.



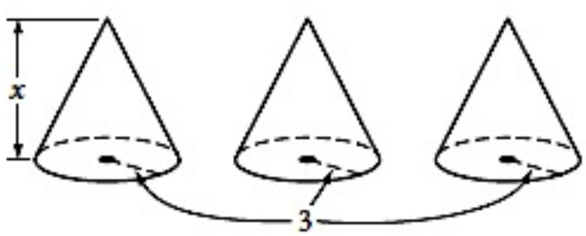
A.



B.



A.



B.